Portfolio Construction using stocks of the Nifty50 companies: A Sharpe’s Single Index Model Approach

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Abstract: Portfolio construction is a complicated task especially for a naive investor. Most of the existing frameworks for portfolio construction use complex mathematical models that may not be easily used by retail investors having limited financial know-how. Sharpe's Single Index Model is an alternative to complex models for Portfolio Construction. It uses Market returns to optimize the portfolio using their historical returns. This paper uses Sharpe's Single Index Model to create an optimum portfolio using stocks of NIFTY50 companies.

Keywords: Portfolio Construction, Stocks, Nifty, Single Index Model

Introduction
The construction of an optimal portfolio has become even more complicated in recent years, as investors expect to maximize returns and minimize risks from their investments. This paper uses Sharpe’s Single Index Model (SIM) to construct an optimal portfolio. It has been preferred over the Markowitz Model as it requires fewer inputs and is easier to calculate. It is named as Single Index Model as it uses only a single index for portfolio construction. In this case, the index chosen by us is NIFTY50. We have constructed a portfolio from NIFTY50 companies in different sectors and have analyzed the portfolio based on risk and return.

Objective of the research:

1. To calculate the risk and return of selected stocks included in NIFTY 50 and analyze each stock's systematic and unsystematic risk.
2. To construct an optimal portfolio using Sharpe’s Single Index model by testing the model on selected stocks listed in NIFTY 50.
3. To calculate the respective proportion for each selected stock to be invested in the portfolio.

Related Work
William Sharpe (1966) proposed a Single Index model which simplifies the process of selecting an optimal portfolio. The Single Index model compares each security returns to the market index. This model reduces the burden of calculations in comparison to Markowitz’s model. Due to the simplicity of this model, many researchers have used Sharpe’s Single Index model to examine optimal portfolio.

Ganesh & Vardharajan (2012) used the Sharpe’s Single Index Model (SIM) to select an optimum portfolio of stocks. The research examined the stocks of six large-cap companies from each of the three major sectors of the Indian Economy i.e. Shipping, Textile and Power. The study found that stocks of only five companies were selected into the optimum portfolio created using the Single
Index Model. Caporin and Lisi (2013) used a conditional Single Index model to study active portfolio management. The research studied the relationship between time-varying values of Betas & Alphas and the historical returns of the stock portfolios. The research modified the SIM and proposed a conceptual model to compare alternative managed portfolios.

Chauhan, A. Apurva. (2014) used Sharpe’s Single Index model to study the returns of the optimal portfolio made using SIM. The research was done on the banking sector stocks listed on the National Stock Exchange. The study found that stocks of 4 companies were used to make the optimum portfolio.

Nandan & Srivastava (2017) conducted a comprehensive study returns of the stocks of individual companies of the Nifty 50 index. The study also used SIM to develop an optimal portfolio and found that only 24 companies out of the 50 companies were selected into the optimal portfolio.

**Methodology**

**Data Source**

The research used historical data of stocks of companies. The adjusted closing price of the stocks was extracted from the website of National Stock Exchange (NSE) and yahoo finance portal. The weekly data of stock prices was collected for a period of five years i.e. 01-01-2016 to 31-12-2020 (261 weeks). The risk free rate is assumed to be 3.70% which is the yield on 364-day treasury bill as on 26th March 2021 (available on RBI website).

**Research Methodology**

This research used the data of ten selected companies from the NIFTY50 index. The major sectors of the Indian economy were represented by these companies and each sector was represented by two companies.

Table 1: Details of companies selected for analysis.

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Company Name</th>
<th>Industry</th>
<th>Symbol</th>
<th>ISIN Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bajaj Auto Ltd.</td>
<td>AUTOMOBILE</td>
<td>BAJAJ-AUTO</td>
<td>INE917I01010</td>
</tr>
<tr>
<td>2</td>
<td>Hero MotoCorp Ltd.</td>
<td>AUTOMOBILE</td>
<td>HEROMOTOCO</td>
<td>INE158A01026</td>
</tr>
<tr>
<td>3</td>
<td>Britannia Industries</td>
<td>CONSUMER GOODS</td>
<td>BRITANNIA</td>
<td>INE216A01030</td>
</tr>
</tbody>
</table>

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Returns

Initially, the weekly returns for the selected stocks and the market index were calculated using the below formula.

\[ Ri = \frac{P_t - P_{t-1}}{P_{t-1}} \]

Where \( Ri \) is the security return

\( P_t \) is the closing price at time \( t \)

\( P_{t-1} \) is the closing price of the stock at time \( t-1 \)

The mean weekly returns of the stocks were calculated by averaging the weekly returns for the five-year period.

The annualized returns were used to compute the C-values for the Sharpe’s Single Index Model.

The annualized returns were calculated using the below formula.
Beta represents the sensitivity of the stock returns with respect to the market returns [6]. The Beta ($\beta$) value of each security was calculated using the below formula

$$\beta = \frac{\text{Covariance}(R_l, R_m)}{\text{Variance}(R_m)}$$

The variance of stock returns and market returns were calculated using MS-Excel functions.

**Systematic Risk**

The systematic risk refers to that portion of the total risk which is caused by factors affecting the prices of all the securities. The source of systematic risk are Economic factors, sociological and political factors.

The systematic risk is calculated using the below formula.

$$\frac{\text{Systematic Risk}}{\text{Total Risk}} = \rho^2$$

Where $\rho$ is the coefficient of correlation between stock returns and market returns.

**Un-Systematic Risk**

The un-systematic risk refers to that portion of the total risk which is caused by factors which are inherent to a particular firm such as lawsuits, labour strike etc.

The un-systematic risk is calculated using the below formula.

$$\frac{\text{Unsystematic Risk}}{\text{Total Risk}} = 1 - \rho^2$$

Where $\rho$ is the coefficient of correlation between stock returns and market returns.

**Characteristic Line**

The characteristic line of a security represents the linear relation between the market returns and the security returns. The formula for characteristic line of a security is

$$R_i = \alpha + \beta \times R_m + e_i$$

Where, $R_i$ is the dependent variable which represents the Return of the security at time interval $i$, $R_m$ is the independent variable which represents the Market return at time interval $i$, $\alpha$ is the intercept and $\beta$ is the slope of the line.
\( \beta \) is the sensitivity of the Security return to the market returns,

\( \alpha \) is the security’s expected excess return when the market excess return is zero

\( e_i \) is the residual value which represents the difference between expected and actual value of return due to firm specific factors.

**Excess Return to Beta Ratio**

The excess returns were calculated by subtracting the risk free rate from the security return. The Excess return to Beta ratio measures the additional return earned for bearing risk per unit and is calculated using the below formula

\[
\text{Excess Return to Beta Ratio} = \frac{R_i - R_f}{\beta}
\]

Where \( R_i \) is the security return

\( R_f \) is the risk free rate

\( \beta \) is the Beta of the stock w.r.t market index

**Sharpe’s Single Index Model**

The formation of an optimal portfolio using Sharpe’s Single Index Model involves the following steps.

1. Calculate the “excess return to beta” ratio for each security.
2. Rank all the securities based on the above ratio (from highest to lowest).
3. Calculate the Cut-off value for each security.

The cut-off value using Sharpe’s Single Index Model is calculated using the below formula.

\[
C = \frac{\sigma_m^2 \sum_{i=1}^{l} \frac{(R_i - R_f)\beta_i}{\sigma_{ei}^2}}{1 + \sigma_m^2 \sum_{i=1}^{l} \frac{\beta_i^2}{\sigma_{ei}^2}}
\]

Where

\( \sigma_m^2 \) = variance in the market index.
\( \sigma_{el}^2 \) = variance of a stock’s movement that is not associated with the movement of the market index; this is the stock’s unsystematic risk.

4. Find the optimal cut-off point (C*, the highest of all) and select all securities upto such cut-off point from Rank 1 onwards.
5. Calculate the proportion (weightage) of each security in the portfolio.

In this step, we first calculate the Zi value for each security using the below formula.

\[
Z_i = \frac{\beta_i^2}{\sigma_{ei}^2} \left[ \frac{R_i - Rf}{\beta_i} - C^* \right]
\]

After this, the weight of a security (Xi) in the portfolio is calculated using the below formula.

\[
X_i = \frac{Z_i}{\sum_{i=1}^{n} Z_i}
\]

**Assumptions underlying Sharpe’s Single Index Model**

1. All investors have homogeneous expectations.
2. A uniform holding period is taken into consideration for determining the risk and return of each security.
3. The price movements of securities are greatly influenced by prevailing economic and business conditions.
4. The indices to which the securities return are correlated are some securities market proxy.

**Results and Discussion**

The results of the analysis are summarized in the following tables.

**Risk and Return**
Table 2: Risk and Returns of stocks and market index.

<table>
<thead>
<tr>
<th>S.No</th>
<th>Name of Stock</th>
<th>Average Daily Stock returns (in %), Ri</th>
<th>Average Daily Market returns (in %) Rm</th>
<th>Variance of daily Stock Returns($\sigma_i^2$)</th>
<th>Variance of Market Returns($\sigma_m^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DRREDDY</td>
<td>0.3061</td>
<td>0.2327</td>
<td>19.2734</td>
<td>5.3477</td>
</tr>
<tr>
<td>2</td>
<td>CIPLA</td>
<td>0.1696</td>
<td>0.2327</td>
<td>15.0248</td>
<td>5.3477</td>
</tr>
<tr>
<td>3</td>
<td>TCS</td>
<td>0.3910</td>
<td>0.2327</td>
<td>11.9302</td>
<td>5.3477</td>
</tr>
<tr>
<td>4</td>
<td>INFOSYS</td>
<td>0.3688</td>
<td>0.2327</td>
<td>13.0150</td>
<td>5.3477</td>
</tr>
<tr>
<td>5</td>
<td>BRITANNIA</td>
<td>0.3933</td>
<td>0.2327</td>
<td>11.9440</td>
<td>5.3477</td>
</tr>
<tr>
<td>6</td>
<td>SBI</td>
<td>0.1902</td>
<td>0.2327</td>
<td>27.3000</td>
<td>5.3477</td>
</tr>
<tr>
<td>7</td>
<td>HEROMOTOCO</td>
<td>0.1356</td>
<td>0.2327</td>
<td>17.7317</td>
<td>5.3477</td>
</tr>
<tr>
<td>8</td>
<td>HDFC</td>
<td>0.4067</td>
<td>0.2327</td>
<td>9.7419</td>
<td>5.3477</td>
</tr>
<tr>
<td>9</td>
<td>ITC</td>
<td>0.0427</td>
<td>0.2327</td>
<td>12.2225</td>
<td>5.3477</td>
</tr>
<tr>
<td>10</td>
<td>BAJAJAUTO</td>
<td>0.1631</td>
<td>0.2327</td>
<td>12.1261</td>
<td>5.3477</td>
</tr>
</tbody>
</table>

From this table, we can see that the HDFC stock has given highest average daily returns during this period. Also, we can see that the SBI stock has the highest variance which implies that the volatility of the returns of the SBI stock is highest.

**Systematic & Unsystematic Risk**

Table 3: Systematic and Unsystematic risk of each stock.
The Beta value of all the stocks except SBI and Hero Motors is less than 1. A beta value of less than 1 means that the returns of these stocks fluctuate to a lesser extent compared to the market index fluctuations. The SBI has a Beta value of 1.50 which means that the stock returns will fluctuate more compared to the fluctuations in the market index. The Beta value of Hero Motor is almost equal to 1 which means that the stock returns move perfectly with the market index returns. The total risk is further segregated into the systematic and unsystematic risk which is given in percentage terms in the above table. We can see that the Dr. Reddy Laboratories has highest unsystematic risk (i.e 91.22%). Similarly, SBI has the highest systematic risk (i.e. 44.37%) among the 10 companies.

**Characteristics Line**

Using the data from above table, we can get the characteristics line equation for all the 10 companies.

Table 4: Characteristic line of each stock.
Sharpe’s Single Index Model Portfolio

**Step 1 and 2** : Calculate the “excess return to beta” ratio for each security. Then, Rank all the securities based on the above ratio (from highest to lowest).

On completing the above two steps we get the following table.

Table 5: Excess returns to Beta ratio for each stock.

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Stock Name</th>
<th>Ri (%) p.a.</th>
<th>Ri-Rf</th>
<th>Beta(Bi)</th>
<th>(Ri-Rf)/Bi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TCS</td>
<td>22.50</td>
<td>18.71</td>
<td>0.6055</td>
<td>30.8999</td>
</tr>
<tr>
<td>2</td>
<td>BRITANNIA</td>
<td>22.64</td>
<td>18.85</td>
<td>0.7008</td>
<td>26.9032</td>
</tr>
<tr>
<td>3</td>
<td>INFOSYS</td>
<td>21.09</td>
<td>17.3</td>
<td>0.6578</td>
<td>26.3052</td>
</tr>
<tr>
<td>4</td>
<td>HDFC</td>
<td>23.49</td>
<td>19.7</td>
<td>0.8241</td>
<td>23.9107</td>
</tr>
<tr>
<td>5</td>
<td>DRREDDY</td>
<td>17.22</td>
<td>13.43</td>
<td>0.5624</td>
<td>23.8850</td>
</tr>
<tr>
<td>S. No</td>
<td>Stock Name</td>
<td>Rf</td>
<td>Beta, Bi</td>
<td>Variance CE ( \frac{R_i - R_f}{\beta} )</td>
<td>( \beta^2 \frac{\sigma_{el}^2}{\sigma_{e_i}^2} )</td>
</tr>
<tr>
<td>-------</td>
<td>--------------</td>
<td>----</td>
<td>----------</td>
<td>---------------------------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>TCS</td>
<td>1</td>
<td>0.6055</td>
<td>30.900</td>
<td>9.97</td>
</tr>
<tr>
<td>2</td>
<td>BRITA NNIA</td>
<td>5</td>
<td>0.7008</td>
<td>26.903</td>
<td>9.32</td>
</tr>
<tr>
<td>3</td>
<td>INFOS YS</td>
<td>0</td>
<td>0.6578</td>
<td>26.305</td>
<td>10.70</td>
</tr>
<tr>
<td>4</td>
<td>HDFC</td>
<td>0</td>
<td>0.8241</td>
<td>23.911</td>
<td>6.11</td>
</tr>
<tr>
<td>5</td>
<td>DRRED DY</td>
<td>3</td>
<td>0.5624</td>
<td>23.885</td>
<td>17.58</td>
</tr>
<tr>
<td>6</td>
<td>CIPLA</td>
<td>5.42</td>
<td>0.5901</td>
<td>9.184</td>
<td>13.16</td>
</tr>
</tbody>
</table>

**Step 3:** Calculate the Cut-off value for each security.

Table 6: Calculation of C value for each stock.
The highest value of C is 15.03 for DRREDDY (S.no. 5).

Therefore, C* = 15.03.

We select the first five securities.

Step 4: Calculating weightage of each security.

We calculate the Zi value and Xi(weight of security) for all five securities using the below formula.

Table 7: Calculation of weights of stocks in the optimum portfolio.

<table>
<thead>
<tr>
<th>S.No</th>
<th>Stock Name</th>
<th>Ri (% p.a.)</th>
<th>Ri-Rf</th>
<th>Bi</th>
<th>(Ri-Rf)/Bi</th>
<th>Variance (σ^2)</th>
<th>Zi</th>
<th>Xi = \frac{zi}{\sum zi}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TCS</td>
<td>22.50</td>
<td>18.71</td>
<td>0.6055</td>
<td>30.8999</td>
<td>9.9695</td>
<td>0.6512</td>
<td>0.1980</td>
</tr>
<tr>
<td>2</td>
<td>BRITANNIA</td>
<td>22.64</td>
<td>18.85</td>
<td>0.7008</td>
<td>26.9032</td>
<td>9.3173</td>
<td>0.7228</td>
<td>0.2198</td>
</tr>
<tr>
<td>3</td>
<td>INFOSYS</td>
<td>21.09</td>
<td>17.30</td>
<td>0.6578</td>
<td>26.3052</td>
<td>10.7009</td>
<td>0.5303</td>
<td>0.1613</td>
</tr>
<tr>
<td>4</td>
<td>HDFC</td>
<td>23.49</td>
<td>19.70</td>
<td>0.8241</td>
<td>23.9107</td>
<td>6.11</td>
<td>1.1914</td>
<td>0.3623</td>
</tr>
<tr>
<td>5</td>
<td>DRREDDY</td>
<td>17.22</td>
<td>13.43</td>
<td>0.5624</td>
<td>23.8850</td>
<td>17.5818</td>
<td>0.1924</td>
<td>0.0585</td>
</tr>
</tbody>
</table>
Based on the above calculations, the optimal portfolio should contain the following proportions of the five security.

Table 8: Optimum Portfolio and stock weights.

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Stock Name</th>
<th>Proportion (Xi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TCS</td>
<td>19.80%</td>
</tr>
<tr>
<td>2</td>
<td>BRITANNIA</td>
<td>21.98%</td>
</tr>
<tr>
<td>3</td>
<td>INFOSYS</td>
<td>16.13%</td>
</tr>
<tr>
<td>4</td>
<td>HDFC</td>
<td>36.23%</td>
</tr>
<tr>
<td>5</td>
<td>DRREDDY</td>
<td>5.85%</td>
</tr>
</tbody>
</table>

The highest proportion in the portfolio is of TCS stocks and the lowest proportion is of Dr. Reddy Laboratories stock.

We can also calculate the return of the optimal portfolio using the historical values of returns for the above five securities.

**Portfolio Return = Σ (Xi * Ri)**

\[
= 22.50 \times 0.1980 + 22.64 \times 0.2198 + 21.09 \times 0.1613 + 23.49 \times 0.3623 + 17.22 \times 0.0585
\]

\[
= 22.36\%
\]

**Conclusion and future Scope**

The construction of an optimal portfolio is a difficult task especially for the retail individual investors. The present study tested the Sharpe’s Single Model Index to generate an optimal portfolio. The stocks in the optimal portfolio are from IT sector, Banking sector and pharma sector. This implies the strong growth and relatively higher stability in returns in these sectors. The total portfolio return is significantly higher and we also get the diversification benefits resulting in reduced risk. The portfolio has the highest weight for HDFC stock. On checking the table, we can see that the HDFC bank has the highest return to risk ratio. Similarly, the lowest weight in the portfolio is of Dr. Reddy laboratories which has the lowest return to risk ratio. However, we need
to note the underlying assumptions in the Sharpe’s Single Index Model. Chandra (2009) found empirical evidence which substantiates that complicated models have not been able to outperform the ability of single index model to predict the covariance existing between security return [9]. The SIM takes into account only one factor but there are conditions beyond market conditions which can impact the securities prices. Thus, this model presents a very simplified picture. However, it is an effective tool for portfolio optimization.

References


