Portfolio Construction using stocks of the Nifty50 companies: A Sharpe's Single Index Model Approach

Sourabh Sharma
Department of Management Studies, IIT Delhi

Abstract : Portfolio construction is a complicated task especially for a naive investor. Most of the existing frameworks for portfolio construction use complex mathematical models that may not be easily used by retail investors having limited financial know-how. Sharpe's Single Index Model is an alternative to complex models for Portfolio Construction. It uses Market returns to optimize the portfolio using their historical returns. This paper uses Sharpe's Single Index Model to create an optimum portfolio using stocks of NIFTY50 companies.

Keywords: *Portfolio Construction, Stocks, Nifty, Single Index Model*

Introduction

The construction of an optimal portfolio has become even more complicated in recent years, as investors expect to maximize returns and minimize risks from their investments. This paper uses Sharpe's Single Index Model (SIM) to construct an optimal portfolio. It has been preferred over the Markowitz Model as it requires fewer inputs and is easier to calculate. It is named as Single Index Model as it uses only a single index for portfolio construction. In this case, the index chosen by us is NIFTY50. We have constructed a portfolio from NIFTY50 companies in different sectors and have analyzed the portfolio based on risk and return.

Objective of the research:

1. To calculate the risk and return of selected stocks included in NIFTY 50 and analyze each stock's systematic and unsystematic risk.
2. To construct an optimal portfolio using Sharpe's Single Index model by testing the model on selected stocks listed in NIFTY 50.
3. To calculate the respective proportion for each selected stock to be invested in the portfolio.

Related Work

William Sharpe (1966) proposed a Single Index model which simplifies the process of selecting an optimal portfolio The Single Index model compares each security returns to the market index. This model reduces the burden of calculations in comparison to Markowitz's model. Due to the simplicity of this model, many researchers have used Sharpe's Single Index model to examine optimal portfolio.

Ganesh & Vardharajan (2012) used the Sharpe's Single Index Model (SIM) to select an optimum portfolio of stocks .The research examined the stocks of six large-cap companies from each of the three major sectors of the Indian Economy i.e. Shipping, Textile and Power. The study found that stocks of only five companies were selected into the optimum portfolio created using the Single

Index Model. Caporin and Lisi (2013) used a conditional Single Index model to study active portfolio management. The research studied the relationship between time-varying values of Betas & Alphas and the historical returns of the stock portfolios. The research modified the SIM and proposed a conceptual model to compare alternative managed portfolios.

Chauhan, A. Apurva.(2014) used Sharpe's Single Index model to study the returns of the optimal portfolio made using SIM. The research was done on the banking sector stocks listed on the National Stock Exchange. The study found that stocks of 4 companies were used to make the optimum portfolio.

Nandan & Srivastava (2017) conducted a comprehensive study returns of the stocks of individual companies of the Nifty 50 index .The study also used SIM to develop an optimal portfolio and found that only 24 companies out of the 50 companies were selected into the optimal portfolio.

Methodology

Data Source

The research used historical data of stocks of companies. The adjusted closing price of the stocks was extracted from the website of National Stock Exchange (NSE) and yahoo finance portal. The weekly data of stock prices was collected for a period of five years i.e. 01-01-2016 to 31-12-2020 (261 weeks). The risk free rate is assumed to be 3.70% which is the yield on 364-day treasury bill as on 26th March 2021(available on RBI website).

Research Methodology

This research used the data of ten selected companies from the NIFTY50 index. The major sectors of the Indian economy were represented by these companies and each sector was represented by two companies.

Table 1: Details of companies selected for analysis.

S.No.	Company Name	Industry	Symbol	ISIN Code
1	Bajaj Auto Ltd.	AUTOMOBILE	BAJAJ-AUTO	INE917I01010
2	Hero MotoCorp Ltd.	AUTOMOBILE	HEROMOTOCO	INE158A01026
3	Britannia Industries Ltd.	CONSUMER GOODS	BRITANNIA	INE216A01030

4	ITC Ltd.	CONSUMER GOODS	ITC	INE154A01025
5	HDFC Bank Ltd.	FINANCIAL SERVICES	HDFCBANK	INE040A01034
6	State Bank of India	FINANCIAL SERVICES	SBIN	INE062A01020
7	Infosys Ltd.	IT	INFY	INE009A01021
8	Tata Consultancy Services Ltd.	IT	TCS	INE467B01029
9	Cipla Ltd.	PHARMA	CIPLA	INE059A01026
10	Dr. Reddy's Laboratories Ltd.	PHARMA	DRREDDY	INE089A01023

Returns

Initially, the weekly returns for the selected stocks and the market index were calculated using the below formula.

$$Ri = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Where Ri is the security return

P_t is the closing price at time t

P_{t-1} is the closing price of the stock at time t-1

The mean weekly returns of the stocks were calculated by averaging the weekly returns for the five-year period.

The annualized returns were used to compute the C-values for the Sharpe's Single Index Model.

The annualized returns were calculated using the below formula.

$$R_{annual} = (1 + R_{weekly})^{52} - 1$$

Beta

Beta represents the sensitivity of the stock returns with respect to the market returns [6]. The Beta (β) value of each security was calculated using the below formula

$$\beta = \frac{\text{Covariance(Ri,Rm)}}{\text{Variance(Rm)}}$$

The variance of stock returns and market returns were calculated using MS-Excel functions.

Systematic Risk

The systematic risk refers to that portion of the total risk which is caused by factors affecting the prices of all the securities. The source of systematic risk are Economic factors, sociological and political factors.

The systematic risk is calculated using the below formula.

$$\frac{Systematic\ Risk}{Total\ Risk} = \rho^2$$

Where ρ is the coefficient of correlation between stock returns and market returns.

Un-Systematic Risk

The un-systematic risk refers to that portion of the total risk which is caused by factors which are inherent to a particular firm such as lawsuits, labour strike etc.

The un-systematic risk is calculated using the below formula.

$$\frac{Unsystematic\ Risk}{Total\ Risk} = 1 - \rho^2$$

Where ρ is the coefficient of correlation between stock returns and market returns.

Characteristic Line

The characteristic line of a security represents the linear relation between the market returns and the security returns .The formula for characteristic line of a security is

$Ri = \alpha + \beta * Rm + ei$

Where, Ri is the dependent variable which represents the Return of the security at time interval i,

Rm is the independent variable which represents the Market return at time interval i,

β is the sensitivity of the Security return to the market returns,

α is the is the security's expected excess return when the market excess return is zero

ei is the residual value which represents the difference between expected and actual value of return due to firm specific factors.

Excess Return to Beta Ratio

The excess returns were calculated by subtracting the risk free rate from the security return. The Excess return to Beta ratio measures the additional return earned for bearing risk per unit and is calculated using the below formula

Excess Return to Beta Ratio = $\frac{R_i - R_f}{\beta}$

Where Ri is the security return

Rf is the risk free rate

β is the Beta of the stock w.r.t market index

Sharpe's Single Index Model

The formation of an optimal portfolio using Sharpe's Single Index Model involves the following steps.

1. Calculate the "excess return to beta" ratio for each security.
2. Rank all the securities based on the above ratio (from highest to lowest).
3. Calculate the Cut-off value for each security.

The cut-off value using Sharpe's Single Index Model is calculated using the below formula.

$$C = \frac{\sigma_m^2 \sum_{i=1}^{i} \frac{(R_i - R_F)\beta_i}{\sigma_{ei}^2}}{1 + \sigma_m^2 \sum_{i=1}^{i} \frac{\beta_i^2}{\sigma_{ei}^2}}$$

Where

$\sigma_m{}^2$= variance in the market index.

σ_{ei}^2 = variance of a stock's movement that is not associated with the movement of the market index; this is the stock's unsystematic risk.

4. Find the optimal cut-off point (C*, the highest of all) and select all securities upto such cut-off point from Rank 1 onwards.
5. Calculate the proportion (weightage) of each security in the portfolio.

In this step, we first calculate the Zi value for each security using the below formula.

$$Z_i = \frac{\beta_i^2}{\sigma_{ei}^2}\left[\frac{Ri - Rf}{\beta_i} - C^*\right]$$

After this, the weight of a security (Xi) in the portfolio is calculated using the below formula.

$$X_i = \frac{Z_i}{\sum_{i=1}^{n} Z_i}$$

Assumptions underlying Sharpe's Single Index Model

1. All investors have homogeneous expectations.
2. A uniform holding period is taken into consideration for determining the risk and return of each security.
3. The price movements of securities are greatly influenced by prevailing economic and business conditions.
4. The indices to which the securities return are correlated are some securities market proxy.

Results and Discussion

The results of the analysis are summarized in the following tables.

Risk and Return

Table 2: Risk and Returns of stocks and market index.

S.No.	Name of Stock	Average Daily Stock returns (in %), Ri	Average Daily Market returns (in %) Rm	Variance of daily Stock Returns(σ_i^2)	Variance of Market Returns (σ_m^2)
1	DRREDDY	0.3061	0.2327	19.2734	5.3477
2	CIPLA	0.1696	0.2327	15.0248	5.3477
3	TCS	0.3910	0.2327	11.9302	5.3477
4	INFOSYS	0.3688	0.2327	13.0150	5.3477
5	BRITANNIA	0.3933	0.2327	11.9440	5.3477
6	SBI	0.1902	0.2327	27.3000	5.3477
7	HEROMOTOCO	0.1356	0.2327	17.7317	5.3477
8	HDFC	0.4067	0.2327	9.7419	5.3477
9	ITC	0.0427	0.2327	12.2225	5.3477
10	BAJAJAUTO	0.1631	0.2327	12.1261	5.3477

From this table, we can see that the HDFC stock has given highest average daily returns during this period. Also, we can see that the SBI stock has the highest variance which implies that the volatility of the returns of the SBI stock is highest.

Systematic & Unsystematic Risk

Table 3: Systematic and Unsystematic risk of each stock.

S.No.	Name of Stock	BETA, β	Correlation Coefficient	Systematic Risk/Total Risk	Unsystematic Risk/Total Risk	Variance (σ_{ei}^2)	Alpha,α
1	DRREDDY	**0.5624**	0.2963	**8.78%**	**91.22%**	17.5818	0.17518
2	CIPLA	**0.5901**	0.3520	**12.39%**	**87.61%**	13.1627	0.03224
3	TCS	**0.6055**	0.4054	**16.43%**	**83.57%**	9.9695	0.25012
4	INFOSYS	**0.6578**	0.4217	**17.78%**	**82.22%**	10.7009	0.21566
5	BRITANNIA	**0.7008**	0.4690	**21.99%**	**78.01%**	9.3173	0.23022
6	SBI	**1.5050**	0.6661	**44.37%**	**55.63%**	15.1874	-0.16000
7	HEROMOTOCO	**1.0039**	0.5513	**30.39%**	**69.61%**	12.3425	-0.09797
8	HDFC	**0.8241**	0.6106	**37.28%**	**62.72%**	6.1100	0.21486
9	ITC	**0.5606**	0.3708	**13.75%**	**86.25%**	10.5417	-0.08772
10	BAJAJAUTO	**0.6664**	0.4425	**19.58%**	**80.42%**	9.7516	0.00800

The Beta value of all the stocks except SBI and Hero Motors is less than 1. A beta value of less than 1 means that the returns of these stocks fluctuate to a lesser extent compared to the market index fluctuations. The SBI has a Beta value of 1.50 which means that the stock returns will fluctuate more compared to the fluctuations in the market index. The Beta value of Hero Motor is almost equal to 1 which means that the stock returns move perfectly with the market index returns. The total risk is further segregated into the systematic and unsystematic risk which is given in percentage terms in the above table. We can see that the Dr. Reddy Laboratories has highest unsystematic risk (i.e 91.22%). Similarly, SBI has the highest systematic risk (i.e. 44.37%) among the 10 companies.

Characteristics Line

Using the data from above table, we can get the characteristics line equation for all the 10 companies.

Table 4: Characteristic line of each stock.

S.No.	Name of Stock	Characteristic Line Equation
1	DRREDDY	**Ri = 0.1752 + 0.5624 * Rm + e**
2	CIPLA	**Ri = 0.0322 + 0.5901 * Rm + e**
3	TCS	**Ri = 0.2501 + 0.6055 * Rm + e**
4	INFOSYS	**Ri = 0.2157 + 0.6578 * Rm + e**
5	BRITANNIA	**Ri = 0.2302 + 0.7008 * Rm + e**
6	SBI	**Ri = -0.1600 + 1.5050 * Rm + e**
7	HEROMOTOCO	**Ri = -0.0980 + 1.0039 * Rm + e**
8	HDFC	**Ri = 0.2149 + 0.8241 * Rm + e**
9	ITC	**Ri = -0.0877 + 0.5606 * Rm + e**
10	BAJAJAUTO	**Ri = 0.008 + 0.66635 * Rm + e**

Sharpe's Single Index Model Portfolio

Step 1 and 2 : Calculate the "excess return to beta" ratio for each security. Then, Rank all the securities based on the above ratio (from highest to lowest).
On completing the above two steps we get the following table.

Table 5: Excess returns to Beta ratio for each stock.

S.No.	Stock Name	Ri (% p.a.)	Ri-Rf	Beta(Bi)	(Ri-Rf)/Bi
1	TCS	22.50	18.71	0.6055	30.8999
2	BRITANNIA	22.64	18.85	0.7008	26.9032
3	INFOSYS	21.09	17.3	0.6578	26.3052
4	HDFC	23.49	19.7	0.8241	23.9107
5	DRREDDY	17.22	13.43	0.5624	23.8850

6	CIPLA	9.21	5.42	0.5901	9.1845
7	BAJAJAUTO	8.84	5.052	0.6664	7.5817
8	SBI	10.39	6.598	1.5050	4.3842
9	HEROMOTOCO	7.30	3.513	1.0039	3.4998
10	ITC	2.25	-1.54	0.5606	-2.7517

Step 3: Calculate the Cut-off value for each security.

Table 6: Calculation of C value for each stock.

S. No.	Stock Name	Ri-Rf	Beta,Bi	$\frac{R_i - R_f}{\beta}$	Variance (σ_{ei}^2)	Mi = $\frac{(R_i - R_f)}{\sigma_{ei}^2}$	Ni = $\frac{\beta^2}{\sigma_{ei}^2}$	ΣMi	ΣNi	C= $\frac{\sigma_m^2 * \Sigma Mi}{1+ \sigma_m^2 * \Sigma Ni}$
1	TCS	18.71	0.6055	30.900	9.97	1.136	0.037	1.136	0.037	5.08
2	BRITANNIA	18.85	0.7008	26.903	9.32	1.418	0.053	2.555	0.089	9.24
3	INFOSYS	17.30	0.6578	26.305	10.70	1.064	0.040	3.618	0.130	11.42
4	HDFC	19.70	0.8241	23.911	6.11	2.658	0.111	6.276	0.241	14.66
5	DRREDDY	13.43	0.5624	23.885	17.58	0.430	0.018	6.706	0.259	15.03
6	CIPLA	5.42	0.5901	9.184	13.16	0.243	0.026	6.949	0.286	14.71

7	BAJAJ AUTO	5.05	0.6664	7.582	9.75	0.345	0.046	7.294	0.331	14.08
8	SBI	6.60	1.5050	4.384	15.19	0.654	0.149	7.948	0.480	11.91
9	HERO MOTO R CO	3.51	1.0039	3.500	12.34	0.286	0.082	8.234	0.562	11.00
10	ITC	-1.54	0.5606	-2.752	10.54	-0.082	0.030	8.152	0.592	10.47

The highest value of C is 15.03 for DRREDDY (S.no. 5).

Therefore, C* = 15.03.

We select the first five securities.

Step 4: Calculating weightage of each security.

We calculate the Zi value and Xi(weight of security) for all five securities using the below formula.

Table 7: Calculation of weights of stocks in the optimum portfolio.

S.No.	**Stock Name**	**Ri (% p.a.)**	**Ri-Rf**	**Bi**	**(Ri-Rf)/Bi**	**Variance (σ_{ei}^2)**	**Zi**	**Xi= $\frac{Zi}{\Sigma Zi}$**
1	TCS	22.50	18.71	0.6055	30.8999	9.9695	0.6512	0.1980
2	BRITANNIA	22.64	18.85	0.7008	26.9032	9.3173	0.7228	0.2198
3	INFOSYS	21.09	17.30	0.6578	26.3052	10.7009	0.5303	0.1613
4	HDFC	23.49	19.70	0.8241	23.9107	6.11	1.1914	0.3623
5	DRREDDY	17.22	13.43	0.5624	23.8850	17.5818	0.1924	0.0585

Based on the above calculations, the optimal portfolio should contain the following proportions of the five security.

Table 8: Optimum Portfolio and stock weights.

S.No.	Stock Name	Proportion (Xi)
1	TCS	19.80%
2	BRITANNIA	21.98%
3	INFOSYS	16.13%
4	HDFC	36.23%
5	DRREDDY	5.85%

The highest proportion in the portfolio is of TCS stocks and the lowest proportion is of Dr. Reddy Laboratories stock.

We can also calculate the return of the optimal portfolio using the historical values of returns for the above five securities.

Portfolio Return = Σ (Xi * Ri)

= 22.50 * 0.1980 + 22.64 * 0.2198 + 21.09 * 0.1613 + 23.49 *0.3623 + 17.22 * 0.0585

= 22.36%

Conclusion and future Scope

The construction of an optimal portfolio is a difficult task especially for the retail individual investors. The present study tested the Sharpe's Single Model Index to the generate an optimal portfolio. The stocks in the optimal portfolio are from IT sector, Banking sector and pharma sector. This implies the strong growth and relatively higher stability in returns in these sectors. The total portofolio return is significantly higher and we also get the diversification benefits resulting in reduced risk. The portfolio has the highest weight for HDFC stock. On checking the table, we can see that the HDFC bank has the highest return to risk ratio. Similarly, the lowest weight in the portfolio is of Dr. Reddy laboratories which has the lowest return to risk ratio. However, we need

to note the underlying assumptions in the Sharpe's Single Index Model. Chandra (2009) found empirical evidence which substantiates that complicated models have not been able to outperform the ability of single index model to predict the covariance existing between security return [9]. The SIM takes into account only one factor but there are conditions beyond market conditions which can impact the securities prices. Thus, this model presents a very simplified picture. However, it is an effective tool for portfolio optimization.

References

Sharpe, W. F. (1963). A simplified model for portfolio analysis. *Management Science*, 9(2), 277-293.

Ganesh & Varadarajan, P(2012), " Construction of Equity Portfolio of Large Cap Companies of Selected Sector in India with reference to the Sharpe Index Model", *International Journal of Physical and Social Sciences*, 2 (1), 37-50.

Caporin, Massimiliano & Lisi, Francesco, 2013. "A Conditional Single Index model with Local Covariates for detecting and evaluating active portfolio management," The North American *Journal of Economics and Finance, Elsevier*, 26(C), pages 236-249.

Chauhan, A. Apurva.(2014). A Study on Usage of Sharpe's Single Index Model In Portfolio Construction With Reference To Cnx Nifty. *Global Journal of Research Analysis*, 3(10), 92-94.

Nandan T, Srivastava N, Construction of Optimal Portfolio Using Sharpe's Single Index Model: An Empirical Study on Nifty 50 Stocks. *J Manag Res Anal* 2017;4(2):74-83
Elton, E. J., Gruber, M. J., & Padberg, M. W. (1976). Simple criteria for optimal portfolio selection. *The Journal of Finance*, 31(5), 1341-1357.

Nalini, R. (2014). "Optimal Portfolio construction using Sharpe's Single Index Model-A study of selected stocks from BSE",*International Journal of Advanced Research in Management and Social Sciences*, 3(12), 72-93.

Mandal, N. (2013). Optimal Portfolio Construction by Using Sharpe's Single Index Model. *Journal of Institute of Public Enterprise*, 36.

Chandra., P. (2009), Investment Analysis and Portfolio Management, 3e, *McGraw-Hill Publishing Company Ltd*, 77-91